Noise in Frequency Dividers

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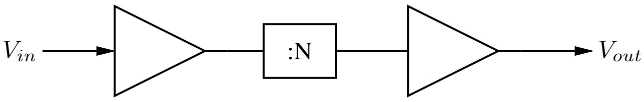

Fig. 1: The circuit model used. The amplifiers are thought to be sine to square-wave converters as in [5, 7].

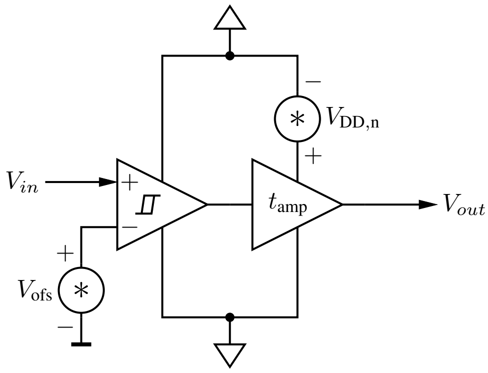

Fig. 2: The circuit model of an sine-to-square wave amplifier stage is simplified to a saturating amplifier input stage with all input related noise being lumped together into the offset voltage V_{ofs}. Noise due to power supply variation $V_{\mathrm{DD,n}}$ is modeled using an noiseless unity-gain buffer stage with a delay of t_{amp} that only depends on the supply voltage

***Abstract*—We calculate the noise performance of Π-, Λ-, and Digital Direct Synthesis (DDS) based dividers. Unlike previous calculations that often rely on ad hoc properties of the signal and its transformation, almost always stated without proof and relying on intuition, we start the calculation from the physical behavior of the circuit and derive its performance directly.**

***Index Terms*—frequency division, noise, flicker noise**

I. Introduction

While there is plenty of literature on low noise frequency division implementations, the literature on how to design them, especially how input and circuit noise affects frequency dividers is pretty slim. The most comprehensive work by Egan [1] which is based on his earlier work in [2] gives very good intuition on a lot of the effects seen and his formulas are based on and match experimental results very well. Unfortunately the source of these effects is not rigorously explained. Later works, like those of Calosso and Rubiola [3], who base their work on Egan, run into similar problems.

In this paper we want to give an ab initio explanation of the circuit behavior, especially how noise from various sources gets transformed within the divider. This work can be seen as a continuation of the work presented in [4] and [5] and is thus is based as well on the input-sensitivity function formalism of [6]. We will focus on digital frequency dividers, namely simple counters (Πdividers), counters with interpolators (Λdividers) and DDS based dividers, as they are the most commonly used today.

II. Circuit Model

We assume that the input V_{in} is sinusoidal, which we first convert to a square-wave with a (chain of) limiting amplifiers similar to that used sine-to-square wave converters as in Collins' work [7] and in [5]. The square-wave is then passed through the divider which generates a square-wave output in case of the Π-divider, a triangle-wave in case of the Λ-divider, and a sinusoid in case of the DDS based divider. The output is then passed again through a limiting amplifier to again generate a square-wave output. This last step is done to ensure that we compare all dividers on a level playing field and do not get artifacts due to variations in the output waveform.

III. Circuit Analysis

A. Previous Results on Amplifier Noise

For completeness, we will quickly repeat the results in [4]. We define phase noise as (c.f. [8, §A.2, pp. 17] and [9, §7.3, pp. 59]):

$$S_\varphi(\xi) := \left\langle \Delta\varphi^2 \right\rangle_f + (2\pi\nu_0)^2 \left\langle \Delta t^2 \right\rangle_f \tag{1}$$

where ν_0 is the signal frequency, $\Delta\varphi$ the variation of the phase of the signal, Δt the variation in delay, and $\left\langle \Delta\varphi^2 \right\rangle_f$ and $\left\langle \Delta t^2 \right\rangle_f$ are the power-spectral average over unity bandwidth:

$$\langle x \rangle_f := \int_{f-1/2}^{f+1/2} \left| \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-2\pi \mathrm{j} \zeta t} \, \mathrm{d}t \right| \mathrm{d}\zeta \tag{2}$$

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Using the impulse sensitivity function (ISF) formalism from Hajimiri and Lee with the resulting phase error due to noise being [6]:

$$\Delta\varphi(t) = \int_{-\infty}^{t} \Gamma(\tau) n(\tau)\, \mathrm{d}\tau \tag{3}$$

where $\Gamma(t)$ is the ISF and $n(t)$ is the noise that affects the system. In a sine-to-square-wave converter the ISF can be approximated as a comb of alternating positive and negative rectangular pulses, we get the Fourier series representation of (c.f. [4, Formulas 14 and 15])

$$\begin{aligned}\Gamma(t) =& \frac{\tau_w}{T_0} \sum_{k=-\infty}^{\infty} \operatorname{sinc}(\pi k \nu_0 \tau_w)\, \mathrm{e}^{-\pi \mathrm{j} k \nu_0 \tau_w}\, \mathrm{e}^{-2\pi \mathrm{j} k \nu_0 t} \\ &+ \frac{\tau_w}{T_0} \sum_{k=-\infty}^{\infty} \operatorname{sinc}(\pi k \nu_0 \tau_w)\, \mathrm{e}^{-\pi \mathrm{j} k \nu_0 \tau_w}\, \mathrm{e}^{-2\pi \mathrm{j} k \nu_0 t}\, \mathrm{e}^{-2\pi \mathrm{j} k \nu_0 \tau_\mathrm{d}}\end{aligned} \tag{4}$$

with ν_0 being the signal frequency, τ_w being the width of the rectangular pulses, τ_d being the phase shift between positive and negative pulses.

Summing over all Fourier components of the ISF leads to a total noise contribution of

$$S_{\varphi,\text{white}}(\xi) \propto \frac{1}{2\pi\nu_0} \left\langle \Delta\varphi^2 \right\rangle_f + 2\pi\nu_0 \left\langle \Delta t^2 \right\rangle_f \tag{5}$$

for white noise and

$$S_{\varphi,\text{flicker,single}}(\xi) \propto \left\langle \Delta\varphi^2 \right\rangle_f + (2\pi\nu_0)^2 \left\langle \Delta t^2 \right\rangle_f \tag{6}$$

$$S_{\varphi,\text{flicker,multi}}(\xi) \propto 2\pi\nu_0 \left\langle \Delta\varphi^2 \right\rangle_f + (2\pi\nu_0)^\alpha \left\langle \Delta t^2 \right\rangle_f \tag{7}$$

$$2 \leq \alpha \leq 3 \tag{8}$$

for flicker noise in single-stage limiting amplifiers and multi-stage amplifiers respectively (c.f. [4, Formulas 22, 23 and 26]).

We would like to note, although we give here the scaling of the phase noise proportional to the contributions of phase and delay noise and their scaling with signal frequency, exact values can be computed by plugging in the values of the harmonics of the ISF and summing over them.

B. Noise Scaling in Frequency Dividers

With this preparation we can finally look how noise gets transformed in the divider itself. We assume the divider being noiseless itself, but instead move its noise contribution either to the input amplifiers, if it is related to the input stage of the divider, or to the output amplifier if it is related to the output stage of the divider. With this in mind, there are two things that affect the output noise of a frequency divider. First we have the spectrum of the output of the divider and how quickly its harmonics decay. Second the division of the input signal itself.

The latter leads to a sampling of noise and thus to an aliasing. This leads to a transformation of variables in the above equations. Namely, the phase gets divided by the division factor N, as is the signal frequency. But any delay does not see any division and stays the same. I.e. we get the following transoform:

$$\Delta\varphi \mapsto \frac{\Delta\varphi}{N} \tag{9}$$

$$\nu_0 \mapsto \frac{\nu_0}{N} \tag{10}$$

$$\Delta t \mapsto \Delta t \tag{11}$$

1) Π-Divider: For the Π-divider, we simply drop all but every N-th edge. Which means we transform the ISF only by the above transform and can simply plug in the new values:

$$S_{\varphi,\text{white}}(\xi) \propto \frac{1}{2\pi\nu_0 N} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{2\pi\nu_0}{N} \left\langle \Delta t^2 \right\rangle_f \tag{12}$$

$$S_{\varphi,\text{flicker,single}}(\xi) \propto \frac{1}{N^2} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{(2\pi\nu_0)^2}{N^2} \left\langle \Delta t^2 \right\rangle_f \tag{13}$$

$$S_{\varphi,\text{flicker,multi}}(\xi) \propto \frac{2\pi\nu_0}{N^3} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{(2\pi\nu_0)^\alpha}{N^\alpha} \left\langle \Delta t^2 \right\rangle_f \tag{14}$$

$$2 \leq \alpha \leq 3 \tag{15}$$

We note that we see the $1/N$ or $10\log(N)$ scaling for white noise, as reported in literature (e.g. [2]). The $1/N^2$ or $20\log(N)$ scaling of flicker noise is more interesting. As far as we are aware of, flicker noise scaling in frequency dividers has not been reported. But multiple references, e.g. [2], report that spurs do scale with $20\log(N)$. Flicker noise and spurs do behave similarly, as the are both effects around the carrier and if they are up- and down-converted add up coherently as the harmonics are originating from the same source and thus all harmonics are correlated.

As the output amplifier, in the case of an Π-divider, has a gain of 1, i.e. it only acts as a buffer and thus as additional stages of a limiting amplifier, i.e. a multi-stage amplifier.

2) Λ-Divider and DDS Based Divider: In the case of the Λ-divider the divider does not drop any of the pulses in the ISF, but scales them. We get first N positive pulses of size $1/N$ and then N negative pulses of size $1/N$. Because we now produce pulses of size $1/N$. This leads to a scaling of the sum of the Fourier components of the ISF by a factor of $1/N$ and thus a scaling of the sampled white noise, additional to the $1/N$ scaling by the reduced frequency. For flicker noise, because of its origin at the baseband and because of its correlated nature, there is no additional gain. The DDS based divider sees the same scaling with ν_0 and N, but due to its output approximating a sine wave gets its white noise contribution scaled down by $\pi/2$.

$$S_{\varphi,\text{white}}(\xi) \propto \frac{1}{2\pi\nu_0 N^2} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{2\pi\nu_0}{N^2} \left\langle \Delta t^2 \right\rangle_f \tag{16}$$

$$S_{\varphi,\text{flicker,single}}(\xi) \propto \frac{1}{N^2} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{(2\pi\nu_0)^2}{N^2} \left\langle \Delta t^2 \right\rangle_f \tag{17}$$

$$S_{\varphi,\text{flicker,multi}}(\xi) \propto \frac{2\pi\nu_0}{N^3} \left\langle \Delta\varphi^2 \right\rangle_f + \frac{(2\pi\nu_0)^\alpha}{N^\alpha} \left\langle \Delta t^2 \right\rangle_f \tag{18}$$

$$2 \leq \alpha \leq 3 \tag{19}$$

IV. Conclusion

We calculated the noise performance of three popular types of frequency dividers. We have derived their behavior directly from their physical behavior without resorting to ad hoc properties of the signal's behavior that are stated without proof as was the case, e.g. in [2]. We also added calculations for the behavior of flicker noise, which was ignored in previous publications.

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